كليات التقنية العليا
Higher Colleges of Technology

Pitch Control of Variable Speed Wind Turbine

ELE 4623 Control Systems

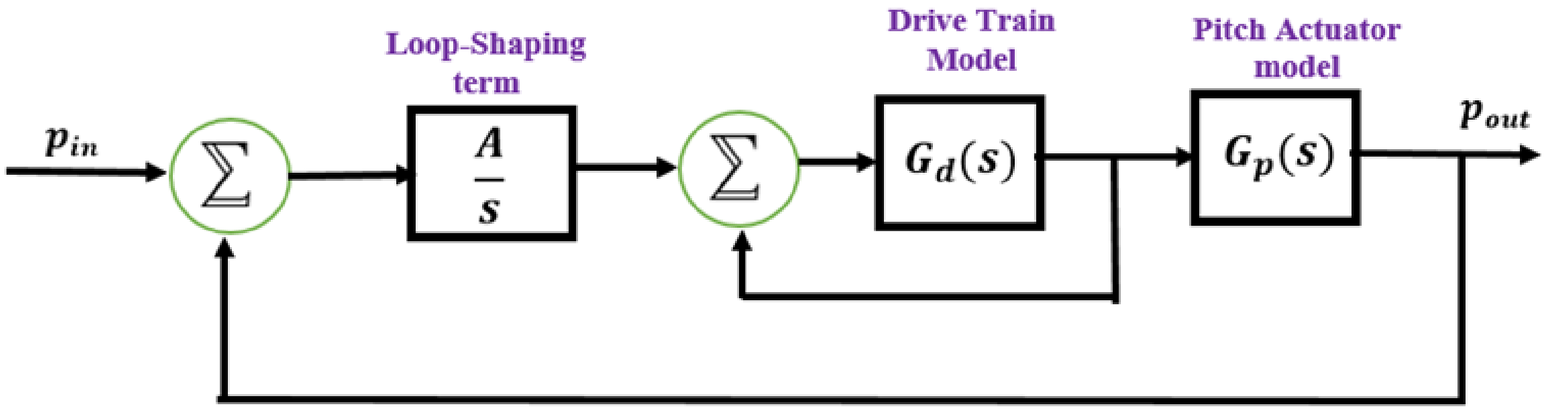
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Introduction & Motivation

Why Pitch Control?

- Wind energy is one of the fastest-growing renewable energy sources globally
- Variable-speed turbines use pitch control to regulate blade angle for efficient energy capture
- Uncontrolled pitch leads to mechanical damage, reduced lifespan, and unsafe structural loading



Project Objective

Design and evaluate classical controllers to achieve **fast, stable, and accurate pitch angle tracking**.

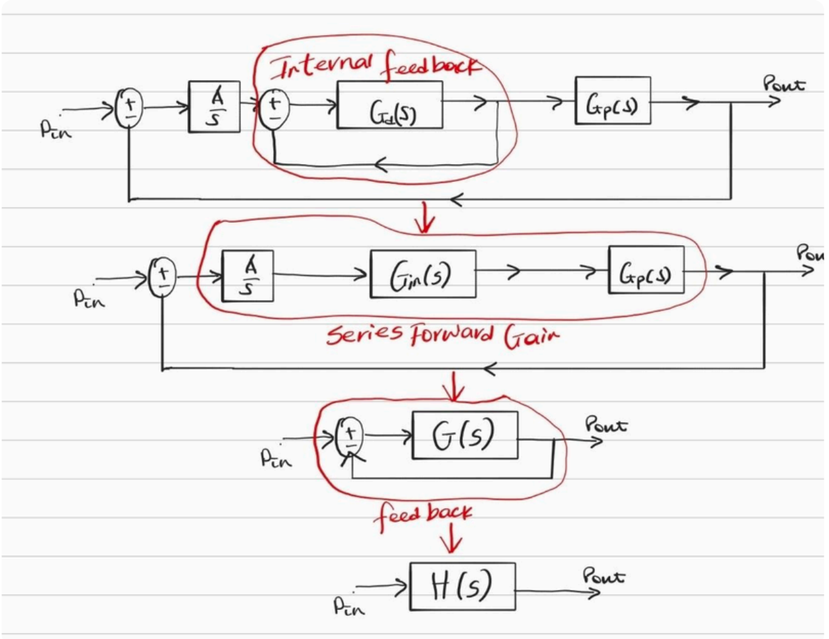
System Architecture

- Pitch actuator **Gp(s)**
- Drive train **Gd(s)**
- Loop-shaping integrator **10/s**
- Unity negative feedback configuration

System Modelling & Transfer Functions

System Parameters

Parameter	Value
Inertia J	1 kg·m²
Damping B	1.5 N·m·rad/s
Time Constant Tp	0.2 s
Gain A	10



Subsystem Transfer Functions

Pitch Actuator

Gp(s) = 1/(0.2s+1) → pole at s = −5

Drive Train

Gd(s) = 1/(s+1.5) → pole at s = −1.5

Inner Feedback

G_inner = Gd/(1+Gd) = 1/(s+2.5) → pole at s = −2.5

Loop-Shaping

K(s) = 10/s → integrator at s = 0

Derived Transfer Functions

G(s) = 50 / [s(s+2.5)(s+5)]

H(s) = 50 / (s^3 + 7.5s^2 + 12.5s + 50)

Original System: Time-Domain Analysis

Closed-Loop Poles

Dominant Pair

$s_{1,2} = -0.375 \pm j2.696$

Third Pole

$s_3 = -6.74$

- We calculated the responses using the dominant poles
- **ζ = 0.138 → OS ≈ 64.7%**
- ~10–15% error due to third-pole exclusion in dominant-pole approximation

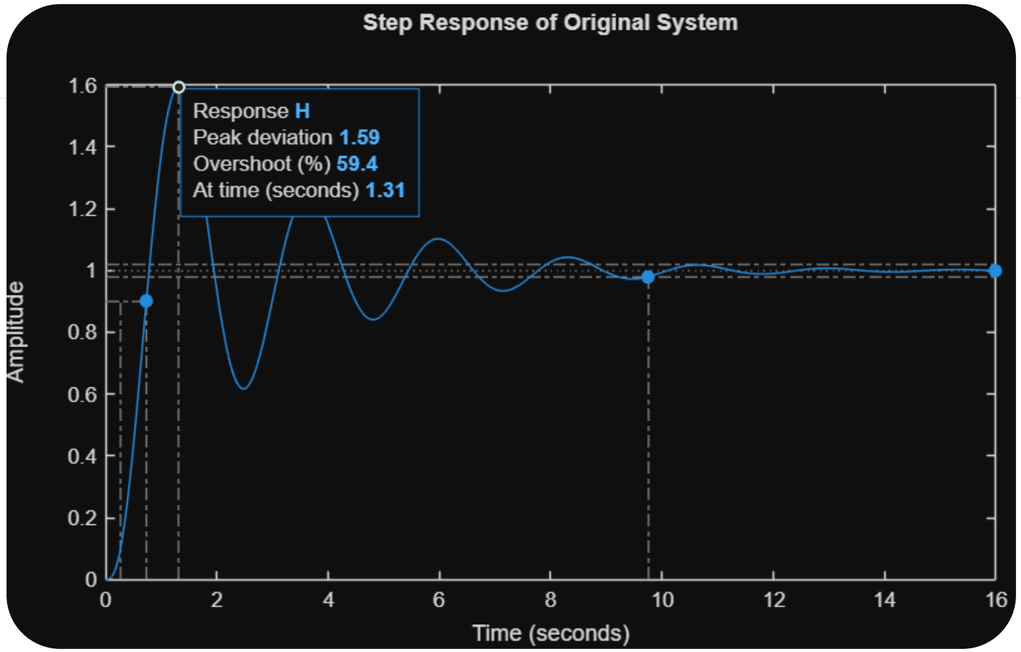
Time-Domain Expression

- $Y(t) = [1 - 0.15e^{-6.75t} - e^{-0.375t}(0.85\cos(2.7t) + 0.5\sin(2.7t))]u(t)$

  Output at t = 0.5s: **Y(0.5) ≈** 0.4315

Analytical vs. Simulation Comparison

Metric	Analytical	Simulation
Overshoot	64.7%	59.4%
Rise Time	0.64 s	0.46 s
Settling Time	10.5 s	9.76 s
Steady-State	1	1

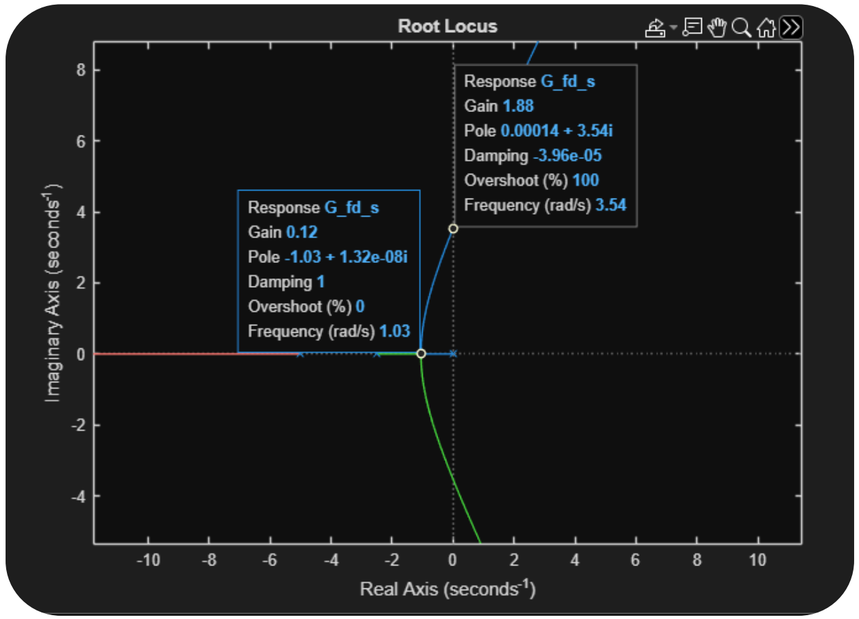


Root Locus Analysis

Open-Loop System

G(s) = 50 / [s(s+2.5)(s+5)]

Open-loop poles at s = 0, −2.5, −5



Breakaway Point

s = −1.06, K = 0.12 → critically damped (ζ = 1)

Critically Stable

s = 0, K = 1.88 → undamped (ζ = 0)

Routh-Hurwitz Table

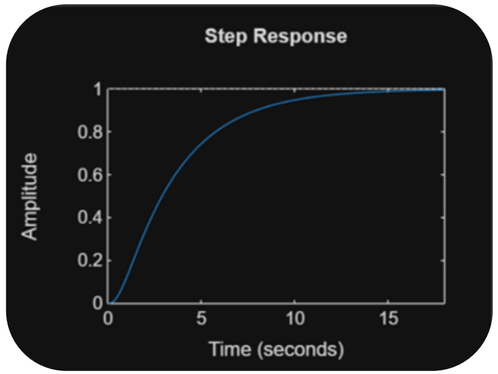
Characteristic equation: $s^3 + 7.5s^2 + 12.5s + 50K = 0$

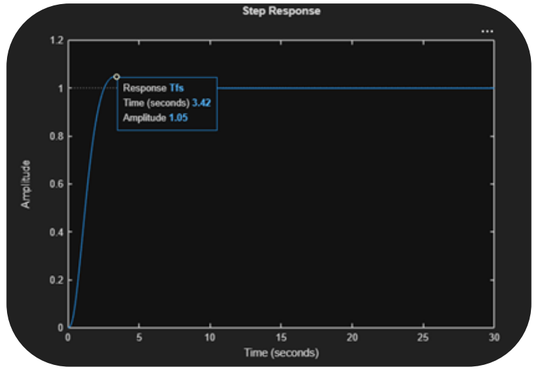
Row	Col 1	Col 2
s^3	1	12.5
s^2	7.5	50K
s^1	(93.75−50K)/7.5	0
s^0	50K	—

- Stability condition confirmed: **0 < K < 1.875**

5% OS Design

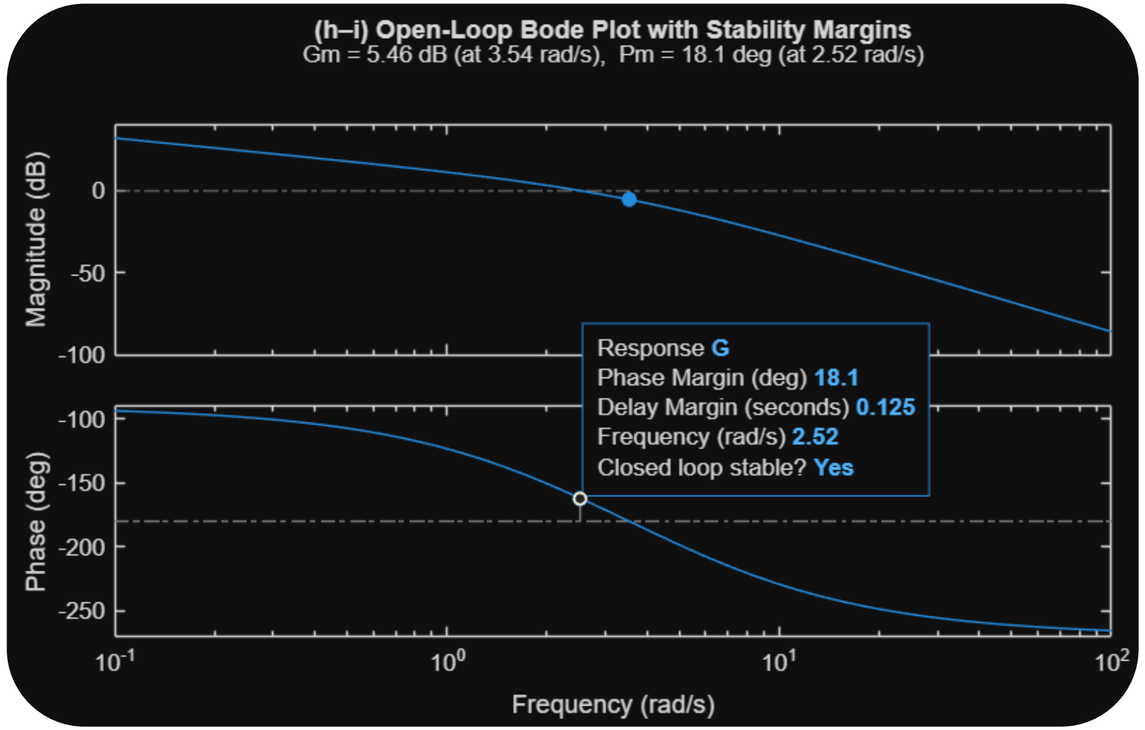
- ζ = 0.69, sgrid intersection → K ≈ 0.066
- The actual 5% OS K = 0.12



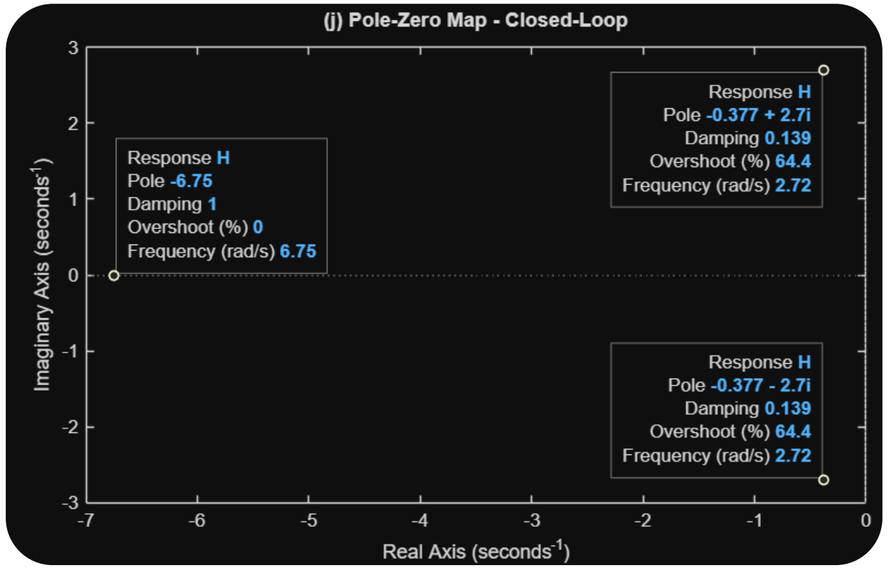


Frequency Domain & Stability Analysis

Bode Plot Results: MATLAB margin()



Stability Verification



→ **Pole Locations**

All poles in Left-Half Plane

Gain Crossover

ωgc = 2.52 rad/s

Phase Crossover

ωpc = 3.54 rad/s

→ **Bode Margins**

PM > 0° and GM > 0 dB → **stable**

Gain Compensator Design

Design Objective

Improve Phase Margin by **10°** (18.1° → 28.1°)

01

Target PM

18.1° + 10° = **28.1°**

02

Target Phase Angle

−180° + 28.1° = **−151.9°**

03

Frequency Lookup

From Bode: phase = −151.9° → **ω ≈ 2.06 rad/s**

04

Magnitude at ω

|G| = +2.86 dB at target frequency

05

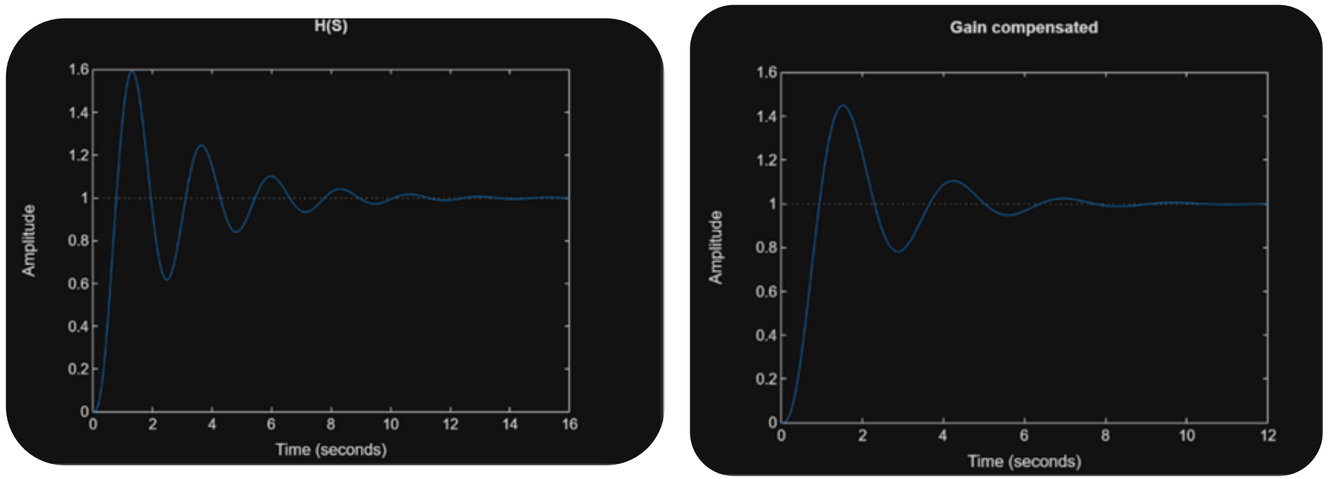
Compensator Gain

K_gc = 10^(−2.86/20) = **0.72**

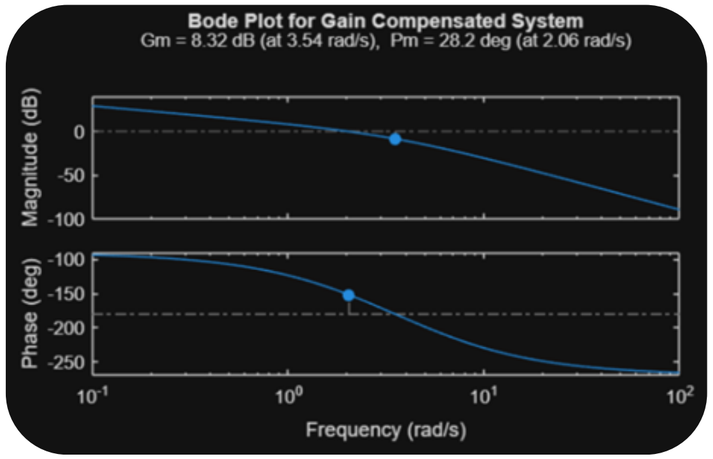
Performance Comparison

Metric	Original	Gain Comp
Rise Time	0.465 s	0.567 s
Settling Time	9.76 s	7.23 s
Overshoot	59.4%	45.1%
Phase Margin	18.1°	28.1°
Gain Margin	5.46 dB	8.26 dB

- Reduced overshoot and improved stability margins achieved with gain compensator.
- Trade-off: Slower rise time due to reduced bandwidth.



original and gain-compensated system step responses



Bode plot of gain-compensated system (PM = 28.1°) and step response comparison Original vs. Gain Comp (MATLAB figure)

Phase-Lead Compensator Design

Design Objective: Achieve Phase Margin = 80° by adding phase at the gain crossover frequency.

Step-by-Step Procedure

01

Required Phase Boost

φ = 80° − 18.1° = **61.9°**

02

Parameter α

α = (1 + sin 61.9°) / (1 − sin 61.9°) = **26.6**

03

Gain Correction

G_dB = −10·log₁₀(26.6) = **−14.25 dB**

04

Time Constant & Compensator

τ = 1/(W_m·√α) → **C_lead(s) = (0.768s + 1)/(0.04802s + 1)**

- Achieved PM = 44.3°, target 80° not met due to crossover shift in single iteration.

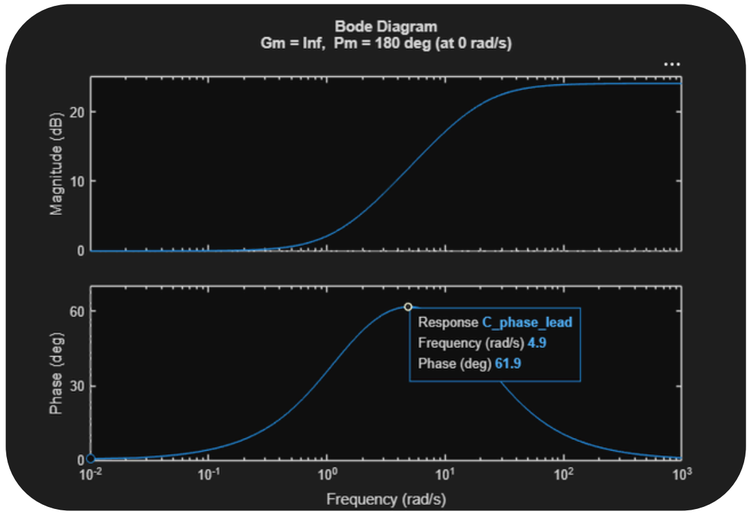


Figure: Bode plot of phase-lead compensator

Results vs. Original System

Metric	Original	Lead
Rise Time	0.465 s	0.258 s
Settling Time	9.76 s	1.46 s
Overshoot	59.4%	20.3%
Phase Margin	18.1°	44.3°
Gain Margin	5.46 dB	12.93 dB

- Best transient speed of all designs, fastest rise and settling times.

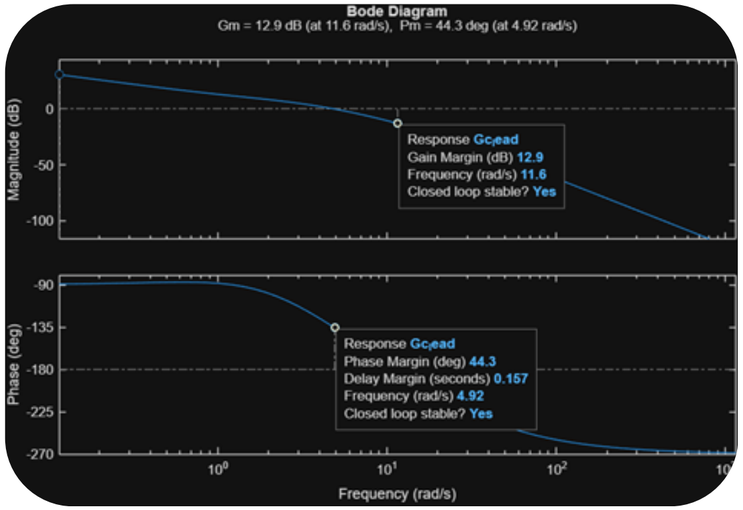


Figure: Bode plot of phase-lead compensated system Gc_lead·G(s) and step response comparison

Phase-Lag Compensator Design

Design Objective: Achieve Phase Margin = 80° through gain attenuation at the crossover frequency.

Step-by-Step Procedure

01

Target Phase

−180° + 80° = **−100°**

02

Find W_s

Locate on Bode plot where phase = −100°

03

Lag Zero Placement

One decade below: τ_lag = 10/W_s

04

Attenuation & Compensator

β = 10^(|G(jW_s)|_dB / 20) → **C_lag(s) = (36.41s + 1)/(526.3s + 1)**

Results vs. Original System

Metric	Original	Lag
Rise Time	0.465 s	5.39 s
Settling Time	9.76 s	60.8 s
Overshoot	59.4%	6.9%
Phase Margin	18.1°	75.3°
Gain Margin	5.46 dB	28.53 dB

- Achieved PM = 75.3°, very close to the 80° target.

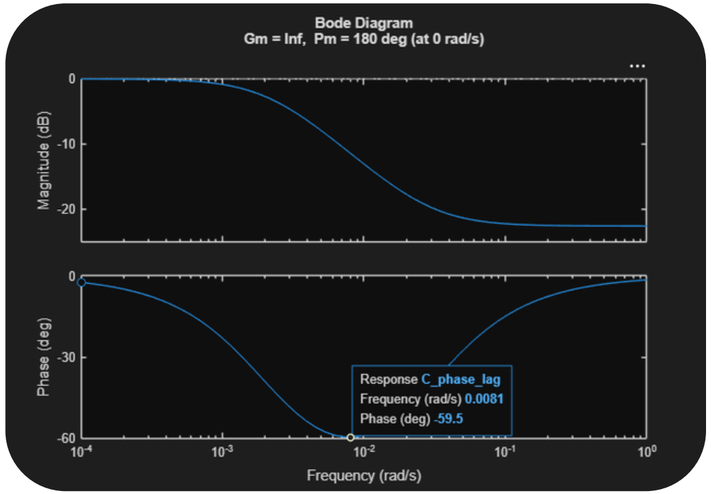


Figure: Bode plot of phase-lag compensater

- Best stability margins and lowest overshoot, but 6× slower settling makes it impractical for real-time wind pitch control.

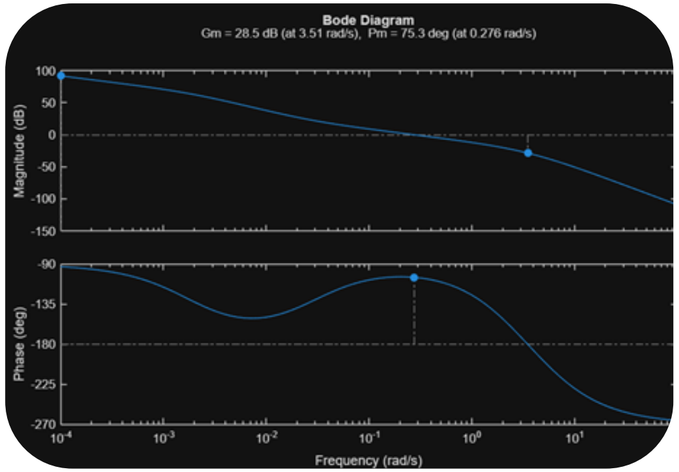


Figure: Bode plot of phase-lag compensated system and step response comparison

PID Controller: Ziegler-Nichols Tuning

Transfer Function: $G_{c_PID}(s) = K_p \cdot (T_d \cdot s^2 + T_i \cdot s + 1)/(T_i \cdot s)$

ZN Tuning Parameters

1.875
Kcr
Critical gain from Routh-Hurwitz

3.54
Wcr (rad/s)
Phase crossover frequency

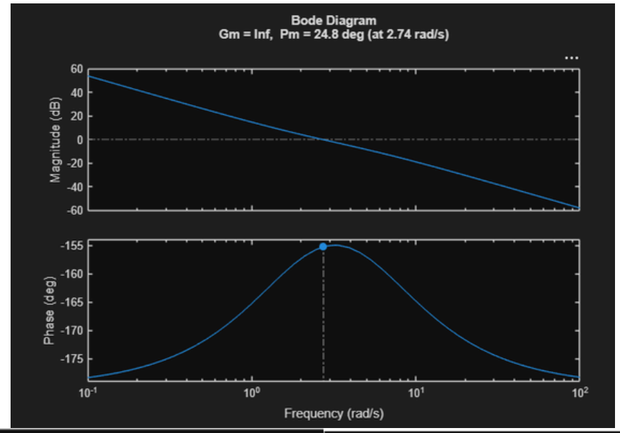
1.775
Pcr (s)
Critical period = 2π/3.54

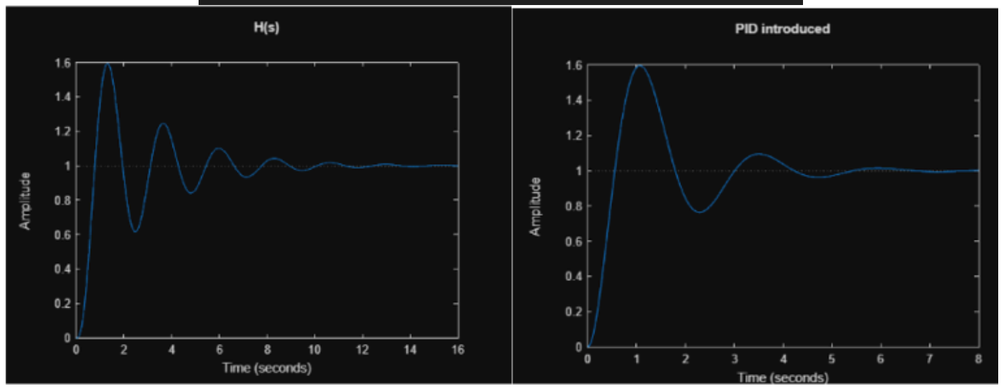
Parameter	Formula	Value
Kp	0.6 × 1.875	1.125
Ti	0.5 × 1.775	0.888 s
Td	0.125 × 1.775	0.222 s
Ki	Kp/Ti	1.267
Kd	Kp×Td	0.250

Achieved PM = 24.8°, GM = ∞. High overshoot is characteristic of ZN tuning, further Kp/Td adjustment would reduce it significantly.

Results vs. Original System

Metric	Original	PID (ZN)
Rise Time	0.465 s	0.375 s
Settling Time	9.76 s	5.14 s
Overshoot	59.4%	59.5%
Phase Margin	18.1°	24.8°
Gain Margin	5.46 dB	∞





Full Performance Comparison

All five designs evaluated side by side across key transient and frequency-domain specifications.

Specification	Original	Gain Comp	Lead	Lag	PID
Rise Time (s)	0.465	0.567	**0.258**	5.39	0.375
Settling Time (s)	9.76	7.23	**1.46**	60.8	5.14
Overshoot (%)	59.4	45.1	20.3	**6.9**	59.5
SS Error	0	≈0	≈0	≈0	0
Phase Margin (°)	18.1	28.1	44.3	**75.3**	24.8
Gain Margin (dB)	5.46	8.26	12.93	28.53	**∞**

Lead

Fastest transient, best for fast blade adjustment applications

Lag

Best stability margins, too slow for real-time wind pitch control

PID

Most balanced, infinite GM, further overshoot tuning needed

Gain Comp

Simplest approach,insufficient as a standalone solution

Theory vs. Simulation & Limitations

Theory vs. Simulation Accuracy

Overshoot Gap

Analytical OS = 64.7% vs. simulated 59.4%, the discrepency is from removing using the dominant pole.

Settling Time

Analytical 10.5 s vs. simulated 9.76 s.

Gain Margin Match

Routh-Hurwitz (1.875) matches MATLAB margin() exactly.

Key Limitations

1. **Dominant-Pole Approximation**

   10–15% error and grows significantly when poles are closer together

2. **ZN PID**

   59.5% overshoot requires iterative fine-tuning beyond initial ZN parameters

3. **Lead Crossover Sensitivity**

   80° target not achieved in one iteration — iterative redesign required

4. **Lag Bandwidth**

   Below 1 rad/s — likely too slow for practical real-time pitch control

Conclusion & Recommendation

The original system is **stable but poorly damped,** PM = 18.1°, OS = 59.4%. All four compensators improved stability and performance, each with distinct trade-offs.

Phase-Lead Fastest response: rise 0.258 s, settling 1.46 s, best for rapid blade adjustment	**Phase-Lag** Best margins: GM = 28.5 dB, OS = 6.9%, too slow for real-time use
PID (ZN) Most balanced: half settling time, infinite GM, needs overshoot tuning	**Gain Comp** Simplest approach, not sufficient as a standalone solution

Recommendation: A tuned PID or refined phase-lead compensator provides the best balance of transient speed, stability robustness, and steady-state accuracy for a system where conditions change rapidly.

